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# Comment and some questions on "Puzzles and the maximum effective moment (MEM) criterion in structural Geology"

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## 1. Introduction

Zheng et al (Zheng and Wang, 2004; Zheng et al., 2011) proposed a new mechanism for ductile formation which is related to **effective moment** instead of shear stress, and the deformation zone develops along plane of **maximum effective moment**. The mathematical expression of maximum effective moment (The criterion of maximum effective moment, simplified as MEM criterion, Zheng and Wang, 2004; Zheng et al., 2011) is that $M_{eff} = 0.5 (\sigma_1 - \sigma_3)$ L sin2$\alpha$sin$\alpha$, where $\sigma_1 - \sigma_3$ is the yield strength of a material or rock, $L$ is the unit length (of cleavage) in the $\sigma_1$ direction, and $\alpha$ is the angle between $\sigma_1$ and a certain plane. The effective moment reaches its maximum value when $\alpha$ is $\pm 54.7^\circ$ and deformation zones tend to appear in pairs with a conjugate angle of 2$\alpha$, 109.4° facing to $\sigma_1$. There is no remarkable $M_{eff}$ drop from the maximum values within the range of $54.7^\circ \pm 10^\circ$, where is favorable for the formation of ductile deformation zone. As a result, the origin of low-angle normal faults, high-angle reverse faults and certain types of conjugate strike-slip faults, which are incompatible with Mohr-Coulomb criterion, can be reasonably explained with MEM criterion (Zheng et al., 2011). Further more, lots of natural and experimental cases were found or collected to support the criterion.

However, in-depth study shows that there is a problem in mechanical derivation in MEM criterion. Actually, only one case, where the direction of the pre-existing cleavage is parallel to $\sigma_1$, is considered in the mechanical derivation (see detail analysis in the next part). If so, MEM criterion can be expanded.

## 2. A problem of mechanical derivation in the MEM criterion

According to the stress analysis figure (Figure 1, Figure 4a in Zheng and Wang, 2004) and the calculation equation of effective moment (equations (3) and (4) in Zheng and Wang, 2004), it can be inferred that pre-existing cleavage is parallel to $\sigma_1$ ("$L$" direction in Fig. 1) in the MEM criterion with the following analysis.

According to the words description ("*$\theta$ is the angle between $\sigma_1$ and the normal to the cleavage and $\alpha$ is the angle between $\sigma_1$ and the cleavage itself*" in the paper (page 276) of Zheng and Wang, 2004) and Fig. 1, the perpendicular direction of "$H$" or potential **ccc** or **ecc** direction in Fig. 1 was defined as the direction of pre-existing cleavage by Zheng and Wang (2004). It is not reasonable and inconsistent with the calculation equation of the effective moment ($M_{eff} = \tau_{eff} H = \tau_\theta L \sin\alpha$, equation (3) in the paper of Zheng and Wang, 2004). Because, if $\theta$ is the angle between $\sigma_1$ and the normal to the cleavage and $\alpha$ is the angle between $\sigma_1$ and the cleavage itself, the following three results are bound to occur: ①The pre-existing cleavage will be parallel to the potential **ccc** or **ecc** direction (Fig. 1). However, if they are parallel, the arm of effective moment of the cleavage will become zero, so will the effective moment. ② $\alpha$ and $\theta$ are not mutually independent. Instead, they are complement to each other (Fig. 1). ③ Once the cleavage direction is given, $\alpha$ (including $\theta$) is a determined value, not a variable. The effective moment calculated with the equation

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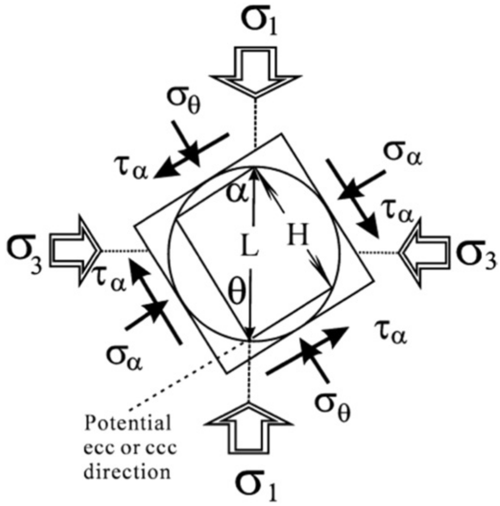


**Fig. 1.** Stress state on the boundary of a unit block with pre-existing cleavage defined by Zheng et al. (adopted from Figure 4a in the paper of Zheng and Wang, 2004). $\sigma_1$ and $\sigma_3$—the maximum and minimum principal stresses; $\theta$—the angle between $\sigma_1$ and the normal to the cleavage; $\alpha$—the angle between $\sigma_1$ and the cleavage itself; $H$—arm; $L$—the side of the unit cube or $H_{max}$ in the $\sigma_1$ direction.

($M_{eff} = 0.5\ (\sigma_1 - \sigma_3)\ L\ \sin2\alpha\sin\alpha$) is also a given value, but not a variable. So, there is no so-called the maximum effective moment value. It is clearly a problem in the definition of angles in the paper, which was likely unintended by Zhen et al.

While, according to the $M_{eff}$ equation and the stress analysis figure (Fig. 1), it can be inferred that the pre-existing cleavage is parallel to $\sigma_1$ direction ("$L$" direction in Fig. 1), $\alpha$ is the angle between potential **ccc** or **ecc** direction and $\sigma_1$ direction and $\theta$ is the complement angle of $\alpha$, which were indicated in Fig. 1. It is not difficult to find that only when pre-existing cleavage is parallel to $\sigma_1$ direction and $\alpha$ is the angle between potential **ccc** or **ecc** direction and $\sigma_1$ ("$L$" direction), the "arm of effective moment" ("$H$") can be equal to $L\sin\alpha$, $\tau_\theta\ (= -\tau_\alpha) = 0.5\ (\ \sigma_1 - \sigma_3\ )\ \sin2\alpha$, and as a result $M_{eff} = 0.5\ (\ \sigma_1 - \sigma_3\ )\ L\ \sin2\alpha\sin\alpha$. At the same time, the "effective moment" ($M_{eff}$) will change with $\alpha$. That means, $M_{eff}$ equation is valid only when pre-existing cleavage is parallel to $\sigma_1$ direction.

Although there are many natural cases where the pre-existing cleavage is parallel to $\sigma_1$ (i.e., horizontal stratum in compressive condition), after all, a pre-existing cleavage and $\sigma_1$ can be in any direction, and the intersect angle between them can be any value in theory. This is a problem in mechanical derivation in the MEM criterion. Zheng and Wang (2004), Zheng et al. (2011) may have not been aware of that the object of effective moment is pre-existing cleavage in mechanical derivation.

## 3. The maximum effective moment for different direction cleavage

We completely follow the idea of Zheng and Wang (2004) to calculate the effective moment ($M_{eff} = \tau_{eff} H$) for different direction cleavage (Fig. 2). As a result,

$$\tau_{eff} = \tau_{\alpha+90^\circ} = 0.5(\sigma_1 - \sigma_3)\sin2(\alpha + 90^\circ) = -0.5(\sigma_1 - \sigma_3)\sin2\alpha \quad (1)$$

$$H = L\sin\delta = L\sin(a - \theta) \quad (2)$$

$$M_{eff} = \tau_{eff}H = -0.5(\sigma_1 - \sigma_3)L\sin2\alpha\sin(\alpha - \theta) \quad (3)$$

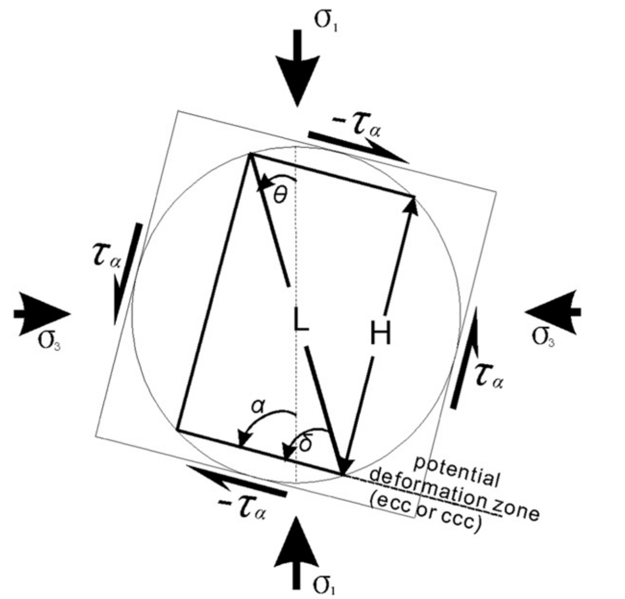


**Fig. 2.** Stress state on a unit block with any given direction cleavage. $\sigma_1$ and $\sigma_3$—the maximum and minimum principal stresses; $\theta$—the angle between $\sigma_1$ and the pre-existing cleavage (counterclockwise as positive), $\alpha$—the angle between $\sigma_1$ and potential deformation zone (**ecc** or **ccc**), and $\delta$—the angle between potential deformation zone and cleavage, $\delta = \alpha - \theta$. $H$—the effective arm of cleavage, $L$—the side of the unit cube (the unit length of cleavage) and the direction of cleavage, and $H = L\sin(\alpha - \theta)$. $\tau_\alpha = 0.5\ (\sigma_1 - \sigma_3)\ \sin2\alpha$.

Where $\theta$ is the angle between the pre-existing cleavage and $\sigma_1$ (counterclockwise as positive), $\alpha$ is the angle between potential deformation zone (**ecc** or **ccc**) and $\sigma_1$, and $\delta$ is the angle between potential deformation zone and cleavage, $\delta = \alpha - \theta$; $(\sigma_1 - \sigma_3)$ is the differential stress when the material yields, $L$ the unit length of the pre-existing cleavage. The definitions of $(\sigma_1 - \sigma_3)$ and $L$ are the same as that in MEM criterion (Zheng and Wang, 2004; Zheng et al., 2011).

When $\theta = 0$ (cleavage parallels to $\sigma_1$), equation (3) becomes the MEM criterion (Zheng and Wang, 2004; Zheng et al., 2011).

We also follow the idea of Zheng and Wang (2004), Zheng et al. (2011) to analyze the maximum value of $M_{eff}$ for different direction cleavage. In order to intuitively reflect the relationship between $M_{eff}$ (that is $\mathbf{\sin2\alpha\sin\ (\alpha - \theta)}$, because $(\sigma_1 - \sigma_3)$ and $L$ are constant for given material, Zheng and Wang (2004), Zheng et al. (2011)) and $\theta$, $\alpha$, Figs. 3 and 4 were made. Fig. 3 is the contour map of $\mathbf{\sin2\alpha\sin}$ $\mathbf{(\alpha - \theta)}$ with variation of $\theta$ and $\alpha$, and Fig. 4 is relationship graph between $\mathbf{\sin2\alpha\sin\ (\alpha - \theta)}$ and $\alpha$ with characteristic orientations at $\theta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$, and $90^\circ$, respectively. As the variation of $\theta$ between 0–90° and 90–180° is symmetrical, the range of $\theta = 0$–$90^\circ$ is discussed here only.

Figs. 3 and 4 show that $M_{eff}$ has, in general, three extreme points for each pre-existing direction ($\theta$) cleavage when $\alpha$ changes between 0° and 180°. However, there are two cases in which two absolute values of the three extreme points are equal and both have a maximum value of 0.77 at $\theta = 0^\circ$ and $90^\circ$ (i.e. pre-existing cleavage parallel or perpendicular to $\sigma_1$). Except for the two cases mentioned above, there is only one extreme point reaches a maximum. That is, there is only one maximum effective moment when $0^\circ < \theta$ s$< 90^\circ$. When $\theta = 45^\circ$, the maximum effective moment reaches the maximum (1.0).

When $\theta = 0^\circ$ (pre-existing cleavage parallel to $\sigma_1$), there are two maximum $M_{eff}$ values (0.77), which occurs symmetrically on two sides of $\sigma_1$ axis with $\alpha = \pm 54.7^\circ$(Figs. 3 and 4, $\alpha = -54.7^\circ$ and $125.3^\circ$ is the same direction), with the same result as MEM criterion (Zheng and Wang, 2004; Zheng et al., 2011). As the pre-existing

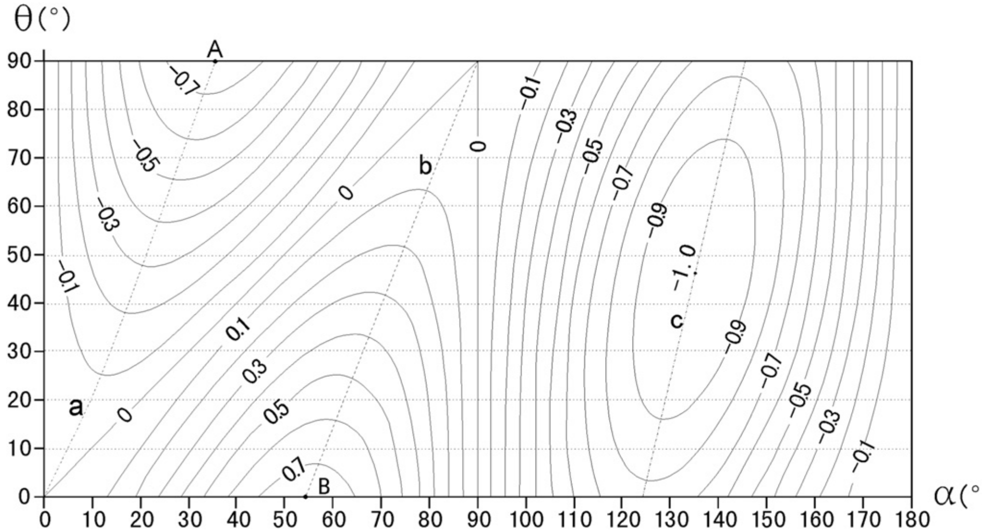


**Fig. 3.** The contour map of $M_{eff}$ ($\sin 2\alpha \sin(\alpha - \theta)$, clockwise rotation as positive and counterclockwise as negative) with different direction of the pre-existing cleavage ($\theta$) and along different potential deformation zone ($\alpha$). There are three $M_{eff}$ extreme points for each given direction cleavage ($\theta$), which constitute curves a, b, c. Among all of the $M_{eff}$ extreme points, the absolute values of point A on curve a ($\theta = 90°$), point B on curve b ($\theta = 0°$), and all points of curve c reach local maximum values for any given $\theta$. That is, there are two maximum $M_{eff}$ value points only at $\theta = 90°$ and $\theta = 0°$; otherwise, there is only one maximum value point.

cleavage deviating from the direction of $\sigma 1$ ($\theta$ increasing from 0° to 90°), there is only one maximum $M_{eff}$ value (0.77–1.0) with $\alpha$ from 54.7° to 35.3° (Figs. 3 and 4). When $\theta = 90°$ (pre-existing cleavage perpendicular to $\sigma_1$), there are also two maximum values of $M_{eff}$, which occur symmetrically on two sides of the $\sigma_1$ axis at $\alpha = \pm 35.3°$ (Figs. 3 and 4).

The above analysis shows that the orientation and assemblage, along which the maximum effective moment appears, are different with the pre-existing cleavage in different directions.

## 4. Some questions on "MEM criterion"

Zheng et al. (2011) have noticed that the $M_{eff}$ will change with unit-length $L$ direction and got the same result as ours when cleavage direction is parallel to $\sigma_3$ ("*…if taking the unit-length in the $\sigma_3$ direction, the maximum value of $M_{eff}$ will appear in the directions of 35.3° to the $\sigma_1$ direction. The conjugate angle predicted in this way should be 70.6°*, …", Page 1396 in Zheng et al., 2011). However, Zheng et al. (2011) may have not noticed that unit-length $L$ direction should be the direction of pre-existing cleavage, and thought the results is invalid (*Although this result is also derived from mathematics, the predicted orientation with respect to $\sigma_1$ departs greatly from observations in nature and experiments* (Fig. 1) *and is, therefore, invalid*. Page 1396 in Zheng et al., 2011). But, this understanding is contradictory with the principle of effective moment proposed by authors themselves. Because, if the principle of effective moment is valid, the results derived with the same principle should be valid, except that the principle is conditional.

But, we still have some questions: ①When the pre-existing cleavage is not parallel to $\sigma_1$, the effective moment does work? ②If it does not work, which mechanism (Mohr-Coulomb rupturing?) does work? ③If it does work, the results of PART 2, in

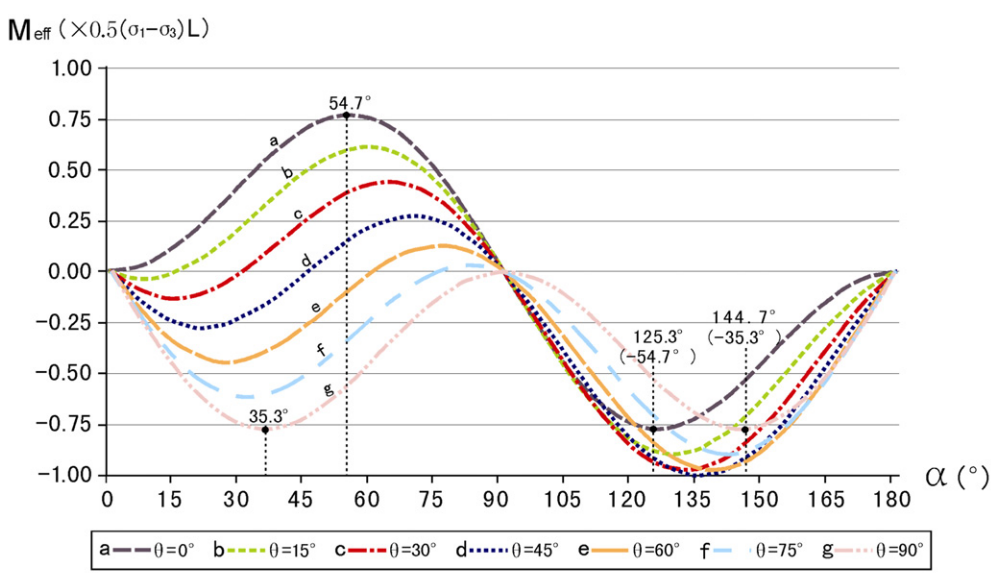


**Fig. 4.** The relationship between $M_{eff}$ ($\sin 2\alpha \sin(\alpha - \theta)$) of the seven characteristic cleavage directions and $\alpha$. There are two $M_{eff}$ maximum values (absolute value) only when $\theta = 90°$ or 0°. In other conditions (0° < $\theta$ < 90°), only one maximum value will occur at 125.3° < $\alpha$ < 144.7°. Curves a, b, c, d, e, f, g represent different directions of the pre-existing cleavage.

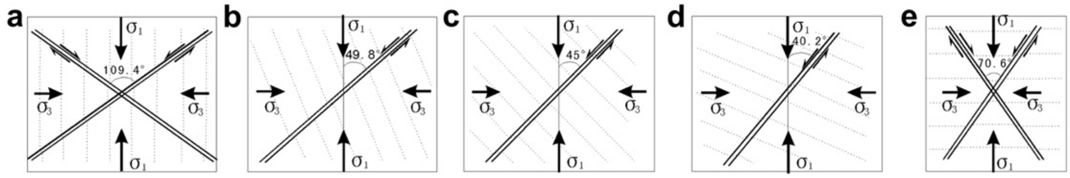


**Fig. 5.** The predicted deformation zone and the shear directions for five characteristic cleavage directions. a) $\theta = 0°$ (pre-existing cleavage parallel to $\sigma_1$); b) $\theta = 22.5°$; c) $\theta = 45°$; d) $\theta = 67.5°$; e) $\theta = 90°$ (pre-existing cleavage perpendicular to $\sigma_1$). Dotted line is the direction of the pre-existing cleavage.

which we completely follow the principle of Zheng and Wang (2004) to calculate the effective moment, are abound to occur. That means, conjugate deformation zones are predicted to occur with different conjugate angle only when the pre-existing cleavage is parallel or perpendicular to $\sigma_1$, otherwise, only one deformation zone is predicted to occur (Fig. 5). But, how to understand the origin of low-angle normal faults, where $\sigma_1$ is normal or sub-normal to pre-existing mylonitic foliation?

If the experiments of Paterson and Weiss (1966) are expanded to any direction for phyllite, the answer may be got.

If the above questions are resolved, it can be determined whether the MEM criterion can be expanded. If the effective moment does work when the pre-existing cleavage is not parallel to $\sigma_1$, the orientation and assemblage of predicted deformation zone are different with the pre-existing cleavage in different directions (Fig. 5).

## 5. Conclusion

The direction of the maximum effective moment, which causes the pre-existing cleavage to deform, is related to the direction of the cleavage. The maximum effective moment can be determined with the equation $M_{\text{eff}} = -0.5\ (\sigma_1 - \sigma_3)\ \text{Lsin}2\alpha\text{sin}\ (\alpha - \theta)$. MEM criterion ($M_{\text{eff}} = 0.5\ (\sigma_1 - \sigma_3)\ \text{Lsin}2\alpha\text{sin}\alpha$) proposed by Zheng and Wang (2004), Zheng et al. (2011) should be conditional: the pre-existing cleavage is parallel to $\sigma_1$.

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